Exam C (Part II) **Name**

Important: Show all your work in the spaces provided. Present your solutions in a well organized, neat, and legible way. Calculators may be used *only* in elementary computational mode and trig mode, and not in calculus mode (even for exploratory purposes).

1. Find all the critical numbers of the function $f(x) = \frac{(x^2-4)^2}{x-3}$. How many of them result in horizontal tangents of the graph? Does the graph have any vertical tangents?

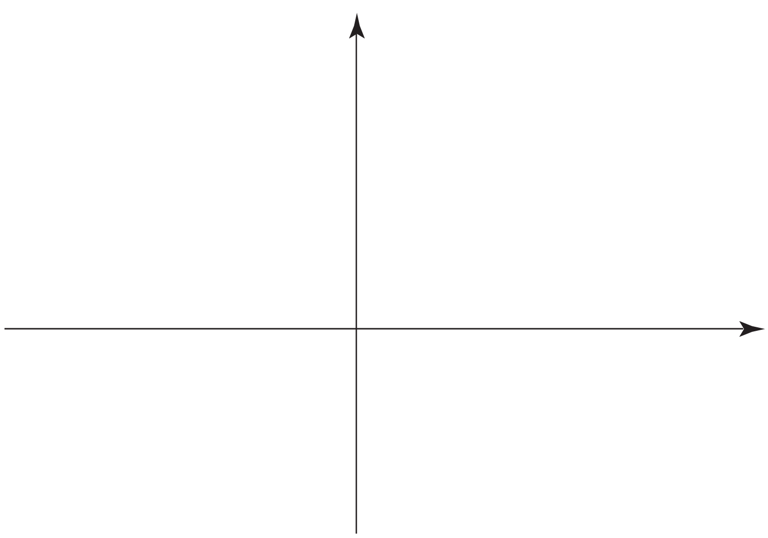
Critical points:

2. Describe in detail three geometric interpretations (with explicit functions) that the definite integral $\int_0^2 \sqrt{1+4x^2}\, dx$ has.

3. Consider the upper half of the circle of radius 1 with center at the origin. For any x with $-1 \leq x \leq 1$, let $G(x)$ be the area under this curve (and above the x-axis) from -1 to x. What is the derivative of the function $G(x)$ equal to?

4. A culture of bacteria is being studied in a lab in a situation of exponential growth. It is known that at time $t = 0$ there were 750 cells in the culture, and that at time $t = 3$ hours, the number of cells was increasing at a rate of 2,250 cells per hour.

4a. Carefully draw the graphs of the functions $y = \frac{9}{x}$ and $y = e^x$ over the interval $1 \leq x \leq 2$ into the xy-coordinate plane provided. Use your sketch to approximate the growth constant λ of the culture.



Ans.

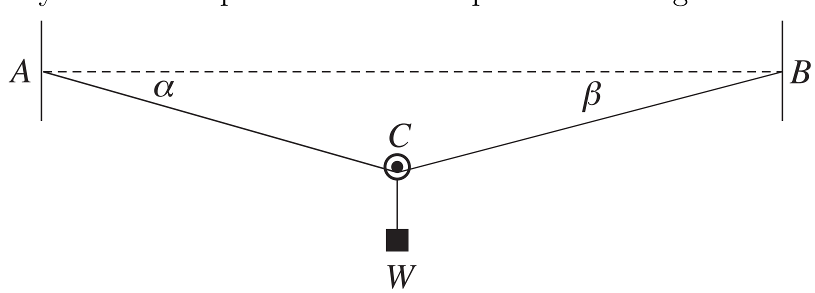
4b. Use your approximation of λ to estimate the number of cells in the culture at time $t = 5$ hours.

5. The radioactive substance Nobelium-253 (named after, you guessed it ...) has a half-life of 97 seconds. A laboratory study considers 10 milligrams of pure Nobelium-253 at time $t = 0$. How many milligrams of Nobelium-253 will remain in the sample after 3 minutes?

5a. Do the mathematics that shows for any radioactive substance what the connection between the half-life h and the decay constant μ is.

5b. Use (a) to solve Problem 4. (The formulas $y(t) = y_0 e^{-\mu t}$ and $y'(t) = -\mu y(t)$ may be relevant.)

6. A rope of length 26 feet is strung between the tops of two vertical poles of the same height. The poles are 24 feet apart. A weight of $W = 100$ pounds is suspended from the rope by a pulley wheel. The system is in equilibrium. It is depicted in the figure below.



6a. Why must the tensions in the two segments of the rope be equal? By balancing their horizontal components, show that the angles α and β equal.

6b. Determine the tension in the rope. [Hint: the use of a 5, 12, 13 right triangle facilitates things.]

Tension =

7. It's laundry day. A clothesline has 60 wet socks (30 pairs) suspended on it, approximately 3 socks per horizontal foot. The 60 socks weigh a total of 15 pounds. The posts supporting the clothesline are 20 feet apart and the sag in the line at its midpoint is 2 feet.

7a. Determine the maximal and minimal tensions in the line.

7b. Determine the angle the line makes at the post with the horizontal.

7c. Determine the horizontal pull of the line on the top of a post.

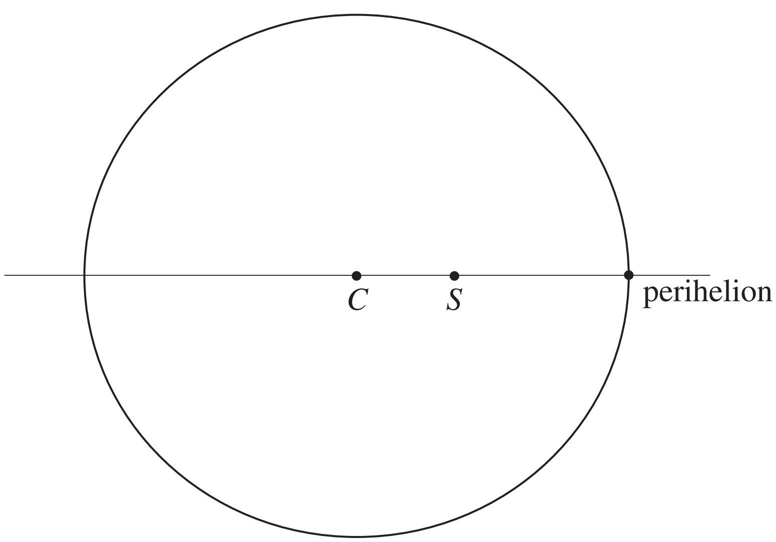
Max Tension:	Min Tension:	Angle:	Pull:

8. Suppose that Mars in its orbit around the Sun S reached its perihelion position exactly 400 days ago. In the problem below you will be asked to determine the precise position P of Mars in its orbit now, in terms of the distance r from P to S in km as well as the angle α between the segment from S to P and that from S to the perihelion position. Work with 3 decimal place accuracy.

8a. Before you begin your computation, estimate the number of steps that the "approximation machine" will require. Provide an explanation for your estimate.

8b. Compute β, r and α. Make use of the data in the table on the last page.

8c. Convert α to degrees and place P carefully into the diagram below.



It should be pointed out that the eccentricity of the orbit of Mars is 0.0934 (the second largest of any planet in the solar system) and that the ellipse of the figure has eccentricity 0.37 (because the distance from the center C to S is about 37% of the distance from C to perihelion). So the orbit of Mars is much more like a circle than the figure above.

Orbital Data of Planets					
Planet	semimajor axis in million km[1]	period of the orbit in years[2]	eccentricity	angle of orbital plane to Earth's	average speed in km/sec[3]
Mercury	57.9092	0.2408	0.2056	7.00°	47.36
Venus	108.2095	0.6152	0.0068	3.39°	35.02
Earth	149.5983	1.0000	0.0167	0.00°	29.78
Mars	227.9438	1.8809	0.0934	1.85°	24.08
Jupiter	778.3408	11.8622	0.0484	1.31°	13.06
Saturn	1426.6664	29.4577	0.0557	2.49°	9.64
Uranus	2870.6582	29.4577	0.0557	2.49°	6.87
Neptune	4498.3964	29.4577	0.0557	2.49°	5.44

1) If the interest is in au, use the conversion 1 au = 149,597,892 km.
2) If the interest is in Earth days, use the conversion 1 year = 365.259636 Earth days.
3) There are (24)(60)(60) = 86,400 seconds.

Some relevant Formulas:

$b = \sqrt{a^2 - c^2} \qquad \varepsilon = \frac{c}{a} \qquad \text{Area} = ab\pi \qquad \kappa = \frac{A_t}{t}$

$x = r\cos\theta,\ \ y = r\sin\theta,\ \ \tan\alpha = \frac{b\sin\beta}{a(\cos\beta - \varepsilon)}$

$r(t) = a(1 - \varepsilon\cos\beta(t)),\ \tan\frac{\alpha(t)}{2} = \sqrt{\frac{1+\varepsilon}{1-\varepsilon}}\tan\frac{\beta(t)}{2}$

$\beta(t) - \varepsilon\sin\beta(t) = \frac{2\pi t}{T},\ \ \beta_1 = \frac{2\pi t}{T},\ \ \beta_{i+1} = \frac{2\pi t}{T} + \varepsilon\sin(\beta_i),\ \ |\beta - \beta_i| \leq \varepsilon^i$

$v(t) = \frac{2\pi a}{T}\sqrt{\frac{2a}{r(t)} - 1}$